Everything on the SAT Math

From learnsatmath.com

Introduction

This document divides everything (yes, *everything*) on the SAT Math into 5 levels based on score. Like all math, the SAT math builds on itself, so you want to understand everything in one level before moving on to the next.

Also note that the score ranges reflect *actual* SAT score, which I estimate is around 20 points lower than Bluebooks 5-6 and 40 points lower than Bluebooks 1-4. So if you get a 650 on Bluebook 1, that would correspond to around a 610 on the actual SAT.

Every formula you see should be *memorized*. Any formulas not explicitly stated are on your provided formula sheet, which I've included at the very end.

And finally, "algebraically" means to solve on paper, and "graphically" means to solve with DESMOS.

Level 1: 200 - 450

1. Algebra

   I've compiled a few basic algebra drills into a Drive folder, but you should feel very comfortable with:

   (a) Order of operations (PEMDAS)

   (b) Fractions

   (c) Factoring

2. DESMOS

   My video on DESMOS covers everything you'll need to know but the most important thing is solving single variable equations (0:54 - 1:53 in the video).

3. Linear Functions

   (a) Functions turn inputs $\rightarrow$ outputs

(b) Understand the relationship between a table, graph, and equation of a linear function

(c) Equation of a line in slope-intercept form

$$y = mx + b$$

(d) Find the slope and y-intercept given 2 points

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

(e) Find the x-intercept of a line algebraically and graphically

Level 2: 450 - 550

1. Variables vs. Constants (what they are)
2. Coefficients (what they are)
3. Systems of equations (use DESMOS)
   (a) What does it mean if a system has a solution?
   (b) What about no solutions?
   (c) What about infinite solutions?
4. Parallel & perpendicular lines
   (a) 2 lines with no solution $\rightarrow$ lines are parallel
   (b) Parallel lines have the same slope
   (c) Perpendicular lines have negative reciprocal slopes
5. Systems of inequalities (use DESMOS)
6. Number of solutions of a polynomial function (based on its graph)
7. Translating English to math
   (a) "of" means multiplication
   (b) "is" means equals
8. Exponent rules
   (a) Exponent product rule

$$x^a * x^b = x^{a+b}$$

(b) Exponent quotient rule

$$\frac{x^a}{x^b} = x^{a-b}$$

(c) Exponent power rule

$$(x^a)^b = x^{a*b}$$

(d) Fractional exponents

$$x^{\frac{1}{a}} = \sqrt[a]{x}$$

(e) Negative exponents

$$x^{-a} = \frac{1}{x^a}$$

(f) Zero power

$$x^0 = 1$$

9. Angle properties (don't memorize by name, but identify them in a figure)
   (a) Complementary angles
   (b) Supplementary angles
   (c) Vertical angles
   (d) Corresponding angles
10. Interior angles for a shape with n angles

$$180(n-2)$$

For example, a triangle has $n = 3$ angles, so $180(3-2) = 180$ interior angles.

11. Isosceles triangles (what they are)
12. Equilateral triangles (what they are)
13. Density

$$density = \frac{mass}{volume}$$

14. Pythagorean theorem (very important!)

$$a^2 + b^2 = c^2$$

15. Sine, Cosine, Tangent (SohCahToa)

$$sin\theta = \frac{Opposite}{Hypotenuse}$$

$$cos\theta = \frac{Adjacent}{Hypotenuse}$$

$$tan\theta = \frac{Opposite}{Adjacent}$$

16. Perimeter & Area (what they are)
17. Volume (just what it is – all the formulas are on your formula sheet)
18. Mean of a data set with n data points

$$\mu = \frac{x_1 + x_2 + ... + x_n}{n}$$
$$\mu = \frac{s}{n} \quad \therefore \quad s = \mu * n$$

19. Midpoint formula (think: mean of 2 points)

$$m = \frac{x_1 + x_2}{2}$$

20. Median of a data set (how to find it)
21. Range of a data set

$$x_{max} - x_{min}$$

Level 3: 550 - 650

1. Integers (what they are)
2. Function translations
   (a) Horizontal shift h units to the *right*

   $$f(x) \to f(x - h)$$

   (b) Vertical shift up k units

   $$f(x) \to f(x) + k$$

3. Percentage *of* vs. percentage *increase* vs. percentage *decrease*

   ex.) 30% of x $\rightarrow 0.3x$
   Increasing x by 30% $\rightarrow 1.3x$
   Decreasing x by 30% $\rightarrow 0.7x$

4. General equation of a circle

$$(x-h)^2 + (y-k)^2 = r^2$$

   Think: Pythagorean theorem applied to a circle with center (h, k) and radius r

5. Find the slope of a line tangent to a circle

6. Arc length

   Let a be arc length, C be circumference, and θ be the *central* angle in degrees.

$$a = \frac{\theta}{360} * C$$

   (It's more intuitive in an example)

7. Sector Area

   Same as arc length but with the circle's area A

$$a = \frac{\theta}{360} * A$$

8. Inscribed angle theorem

   (a) Consider the case where the central angle is 180°

9. Converting degrees $\leftrightarrow$ radians

$$d = r * \frac{180}{\pi}$$

$$r = d * \frac{\pi}{180}$$

10. Congruent triangles

11. Similar triangles

    (a) Angles are the same
    (b) Sides are proportional
    (c) Therefore, trig ratios (sin, cos, tan) are the same

(d) Proven with AA, SSS, SAS, but *not* SSA

12. Conditional probability
13. Box plots
14. Scatterplots
    (a) How to interpret a scatterplot
    (b) How to find line of best fit
15. Sample & population (what makes a sample representative?)
16. Quadratics with DESMOS
    (a) Find solutions of a quadratic
    (b) Find the vertex of a quadratic
17. Factoring a quadratic & finding solutions algebraically

Level 4: 650 - 730

1. Standard form of a quadratic

$$ax^2 + bx + c = 0$$

    (a) a determines upward / downward shape
    (b) c is the y-intercept

2. Vertex form of a quadratic

$$y = a(x - h)^2 + k$$

    (a) a is the same a as standard form
    (b) (h, k) is the vertex

3. Quadratic's vertex given x-intercepts (think: midpoint)

$$h = \frac{x_1 + x_2}{2}$$

4. Quadratic's vertex given a & b

$$h = \frac{-b}{2a}$$

5. Sum of a quadratic's solutions (notice how it comes from 3 and 4)

$$x_1 + x_2 = \frac{-b}{a}$$

6. Product of a quadratic's solutions

$$x_1 * x_2 = \frac{c}{a}$$

7. Find where a quadratic intersects a horizontal line (using DESMOS)
8. Identifying the graph of a polynomial function
9. Exponential functions
   (a) Construct an exponential function given an example
   (b) Find the y-intercept
   (c) How does shifting the exponent change the interpretation? (ex. $100(1.04)^{x-1}$)
   (d) How does scaling the exponent change the interpretation? (ex. $100(1.04)^{\frac{x}{2}}$)
10. Algebra problems with radicals and fractions (like this one)
11. Points in terms of real numbers (like this)
12. Sin & Cosine relationship

$$sin\theta = cos(90 - \theta)$$
$$cos\theta = sin(90 - \theta)$$

13. Square units

    Let u_1 be one type of unit (ex. feet) and u_2 be another type of unit (ex. inches).

$$u_1 = k * u_2$$
$$u_1^2 = k^2 * u_2^2$$

    For example, if 1 foot = 12 inches, then 1 square foot = 144 square inches.

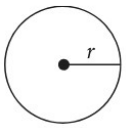
14. Scaling up units (ex. how does doubling side lengths affect area or volume?)
15. Surface area (don't memorize formulas–construct them)
16. Mean = median in a symmetric data set
17. Comparing standard deviations (but NOT computing them)
18. Margin of error
    (a) Margin of error is the expected BUT NOT GUARANTEED deviation of the population mean from the sample mean. This distinction is subtle.
    (b) Margin of error is reduced by increasing sample size

Level 5: 730 - 800

1. Similar triangles created from a triangle's altitude
2. Pyramids
   (a) Surface area of a square right pyramid
   (b) Height of a pyramid $\neq$ slant height
3. Integer factors (like this)
4. Constant-proofing
   (a) Quadratic formula
   $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
   (b) Discriminant of a quadratic for number of solutions
   $$d = b^2 - 4ac$$
   (c) Completing the square (for circle problems)
   $$y = x^2 + bx + c$$
   $$y = (x + \frac{b}{2})^2 + c - (\frac{b}{2})^2$$
5. Nice to know
   (a) Slope of a line from standard form
   $$Ax + By = C \rightarrow m = -\frac{A}{B}$$
   (b) Arc length a where θ is in *radians*
   $$a = r\theta$$
   (c) 3/4/5, 5/12/13 Triangles
   (d) Triangle inequality theorem
6. Mean problems with the term "integer" in them
7. Exponential functions with complicated fractional exponents
8. Quadratics problems solving for some combination of a, b, and c
9. Review your weakest problems and create variations of them. Think: How can College Board make this *even* harder?

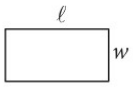
Bluebook Formula Sheet

Some of these formulas were excluded from the first 3 sections, since you can access them easily during the test.

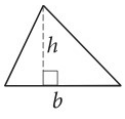


$A = \pi r^2$

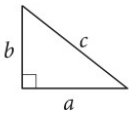
$C = 2\pi r$



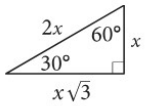
$A = \ell w$

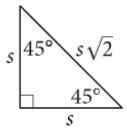


$A = \frac{1}{2}bh$

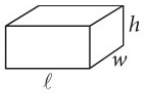


$c^2 = a^2 + b^2$

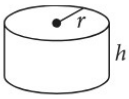




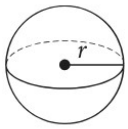
Special Right Triangles



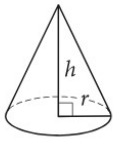
$V = \ell wh$



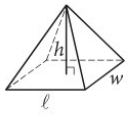
$V = \pi r^2 h$



$V = \frac{4}{3}\pi r^3$



$V = \frac{1}{3}\pi r^2 h$



$V = \frac{1}{3}\ell wh$

The number of degrees of arc in a circle is 360.

The number of radians of arc in a circle is 2π.

The sum of the measures in degrees of the angles of a triangle is 180.